ARMY RESEARCH LABORATORY

Modeling and Simulation of a Differential Roll Projectile

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Abstract

This report develops the equations of motion for a differential roll projectile configuration with seven degrees of freedom. The dynamic equations are generated generically such that the forward and aft components are mass unbalanced. A hydrodynamic bearing exists between the forward and aft components, which couples the roll degree of freedom. A simulation investigation shows that bearing resistance and forward/aft body mass ratio are the dominant factors in determining the roll dynamics. For spin rates typical of fin-stabilized projectiles, the trajectory is essentially independent of both bearing resistance and mass ratio.

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1. Introduction

Compared to conventional munitions, smart munitions involve more design requirements due to additional sensors and control mechanisms. These additional components must seek to minimize the weight and space impact on the overall projectile design so that desired target effects can still be achieved with the weapon. The inherent design conflict between standard projectile design considerations and new requirements imposed by sensors and control mechanisms has led designers to consider more complex geometric configurations. One such configuration is the differential roll projectile. This projectile configuration is comprised of forward and aft components. The forward and aft components are connected through a bearing, which allows the forward and aft portions of the projectile to spin at different rates. Figure 1 shows a schematic of the differential roll projectile configuration.

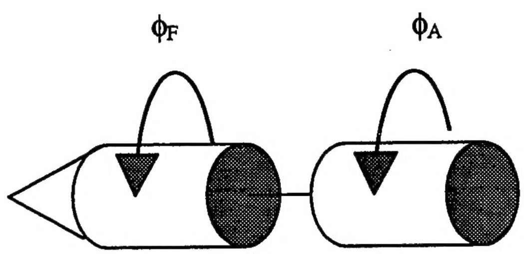

Figure 1. Differential Roll Projectile Schematic.

Typical flight mechanic analysis of a conventional, single-body, munition models the projectile with six degrees of freedom. Dynamic modeling of a differential roll projectile adds an additional roll degree of freedom to the equations of motion. This report begins with the development of a dynamic model of a differential roll projectile in atmospheric flight, including the additional roll degree of freedom. The model is derived such that both the forward and aft bodies can be mass unbalanced. A hydrodynamic bearing couples the forward and aft components in the roll axis. The mathematical model is utilized to show trends in system response as a function of mass ratio and bearing resistance.

2. Differential Roll Projectile Dynamic Model

The mathematical model describing the motion of the differential roll projectile allows for three translation and four rotation rigid-body degrees of freedom. The translation degrees of freedom are the three components of the mass center position vector. The rotation degrees of freedom are the Euler yaw and pitch angles, as well as the forward body roll and aft body roll angles. The equations presented here use the ground surface as an inertial reference frame [1].

Development of the kinematic and dynamic equations of motion is aided by the use of an intermediate reference frame. The sequence of rotations from the inertial frame to the forward and aft bodies consists of a set of body-fixed rotations that are ordered: yaw, pitch, and forward/aft body roll. The fixed-plane reference frame is defined as the intermediate frame before roll rotation. The fixed-plane frame is common to both the forward and aft bodies.

Equation (1) is the translation kinematic differential equations that relate time derivatives of the mass center position components to the mass center velocity components in fixed-plane reference frame.

$$\begin{Bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{Bmatrix} = \begin{bmatrix} c_\theta c_\psi & -s_\psi & s_\theta c_\psi \\ c_\theta s_\psi & c_\psi & s_\theta s_\psi \\ -s_\theta & 0 & c_\theta \end{bmatrix} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix}. \tag{1}$$

Equation (2) is the rotation kinematic differential equations that relate time derivatives of the Euler angles with angular velocity components in the fixed-plane reference frame.

$$\begin{Bmatrix} \dot{\phi}_F \\ \dot{\phi}_A \\ \dot{\theta} \\ \dot{\psi} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_\theta \\ 0 & 1 & 0 & t_\theta \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1/c_\theta \end{bmatrix} \begin{Bmatrix} p_F \\ p_A \\ q \\ r \end{Bmatrix}. \tag{2}$$

Equation (3) is the translation kinetic differential equations described in the fixed-plane reference frame.

$$\begin{Bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{Bmatrix} = \begin{Bmatrix} \dfrac{X}{m} \\ \dfrac{Y}{m} \\ \dfrac{Z}{m} \end{Bmatrix} - \begin{bmatrix} 0 & -r & q \\ r & 0 & 0 \\ -q & 0 & 0 \end{bmatrix} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix}. \tag{3}$$

Equation (4) is the rotation kinetic differential equations described in the fixed-plane reference frame.

$$\begin{Bmatrix} \dot{p}_F \\ \dot{p}_A \\ \dot{q} \\ \dot{r} \end{Bmatrix} = [I]^{-1} \begin{Bmatrix} g_{F1} - M_V \\ g_{A1} + M_V \\ g_{F2} + g_{A2} \\ g_{F3} + g_{A3} \end{Bmatrix}. \tag{4}$$

A derivation of equation (4), along with definitions of the right side components, is provided in Appendices A and B.

As shown in equation (5), the total applied force on the complete configuration is provided by the weight of both the forward and aft bodies ($_W$) and air loads ($_A$).

$$\begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} = \begin{Bmatrix} X_W \\ Y_W \\ Z_W \end{Bmatrix} + \begin{Bmatrix} X_A \\ Y_A \\ Z_A \end{Bmatrix}. \tag{5}$$

The weight portion of the external loads is given by equation (6),

$$\begin{Bmatrix} X_W \\ Y_W \\ Z_W \end{Bmatrix} = mg \begin{Bmatrix} -s_\theta \\ 0 \\ c_\theta \end{Bmatrix}, \tag{6}$$

while the aerodynamic force contribution is given by equation (7),

$$\begin{Bmatrix} X_A \\ Y_A \\ Z_A \end{Bmatrix} = q_a D \begin{Bmatrix} C_{X0} + C_{XA2}\alpha^2 + C_{XB2}\beta^2 \\ C_{Y0} + C_{YB1}\beta \\ C_{Z0} + C_{ZA1}\alpha \end{Bmatrix}. \tag{7}$$

The longitudinal and lateral aerodynamic angles of attack are computed using equation (8).

$$\alpha = \tan^{-1}\left(\frac{w}{u}\right) \text{ and } \beta = \tan^{-1}\left(\frac{v}{u}\right). \tag{8}$$

The aerodynamic coefficients in equation (7) are functions of local Mach number at the projectile mass center. They are computed using linear interpolation from a table of data. The aerodynamic forces and moments are assumed to act solely on the forward body.

The right side of the rotation kinetic equations contains the externally applied moments on both the forward and aft bodies. The external moment components on the forward body are given by equation (9) and contain contributions from steady ($_{SA}$) and unsteady ($_{UA}$) aerodynamics.

$$\begin{Bmatrix} L_A \\ M_A \\ N_A \end{Bmatrix} = \begin{Bmatrix} L_{SA} \\ M_{SA} \\ N_{SA} \end{Bmatrix} + \begin{Bmatrix} L_{UA} \\ M_{UA} \\ N_{UA} \end{Bmatrix}. \tag{9}$$

The steady body aerodynamic moment is computed by a cross between the distance vector from the center of gravity to the center of pressure, and the steady body aerodynamic force vector. The

The unsteady body aerodynamic moment provides a damping source for projectile angular motion and is given by equation (10).

$$\begin{Bmatrix} L_{UA} \\ M_{UA} \\ N_{UA} \end{Bmatrix} = q_a D^2 \begin{Bmatrix} C_{DD} + \dfrac{p_F D C_{LP}}{2V} \\ \dfrac{q D C_{MQ}}{2V} \\ \dfrac{r D C_{NR}}{2V} \end{Bmatrix}. \tag{10}$$

Air density is computed using the center of gravity position of the projectile using the standard atmosphere [2].

3. Simulation Example

In order to exercise the math model discussed previously, consider a 6-ft long, 120-lb projectile. The forward body is fin stabilized and the aft body is an internal circular cylinder. Aerodynamic forces and moments act on the forward body only. For this simulation set, initial forward body velocity is 750 m/s and initial gun elevation is 45°. All other states variables are initially equal to 0. The projectile fins are canted slightly to provide a slowly rolling projectile in steady state.

Figures 2–11 show the state variables of the system vs. time for the conditions mentioned above. The mass ratio of the aft body to the forward body is 1%. Under these circumstances, the projectile has a range of approximately 18 km. Cross range, yaw angle, side velocity, vertical velocity, pitch rate, yaw rate, and aerodynamic angle of attack remain small throughout the event. Pitch attitude steadily decreases from 45° to just below −60° at impact. Figures 2–11 remain the same, independent of the bearing resistance coefficient. Figures 12 and 13 show the roll angle and roll rate response as a function of bearing resistance coefficient. Values of bearing resistance coefficient are 0.000001, 0.000005, 0.00001, 0.00005, 0.0001, 0.0005, 0.001, 0.005,

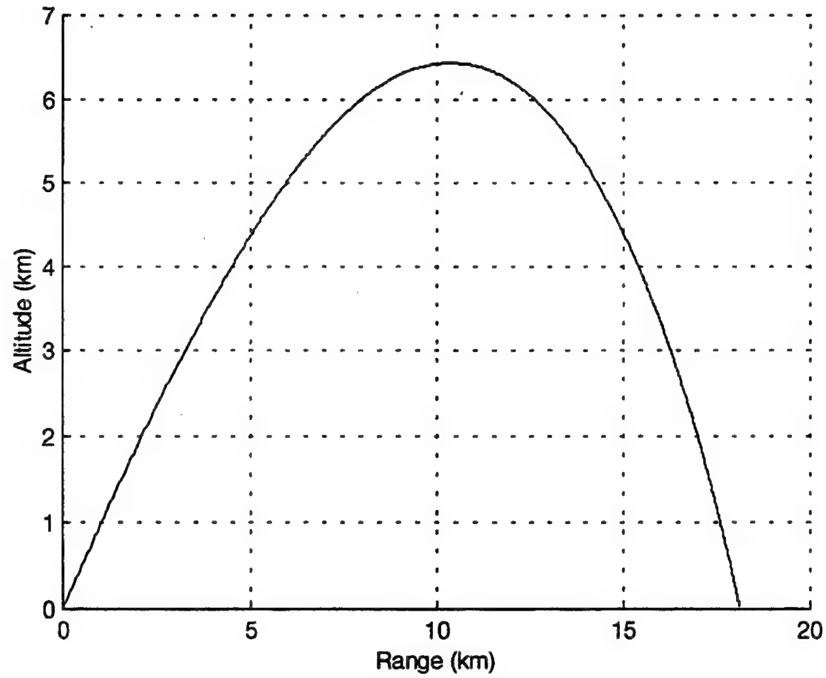

Figure 2. Range.

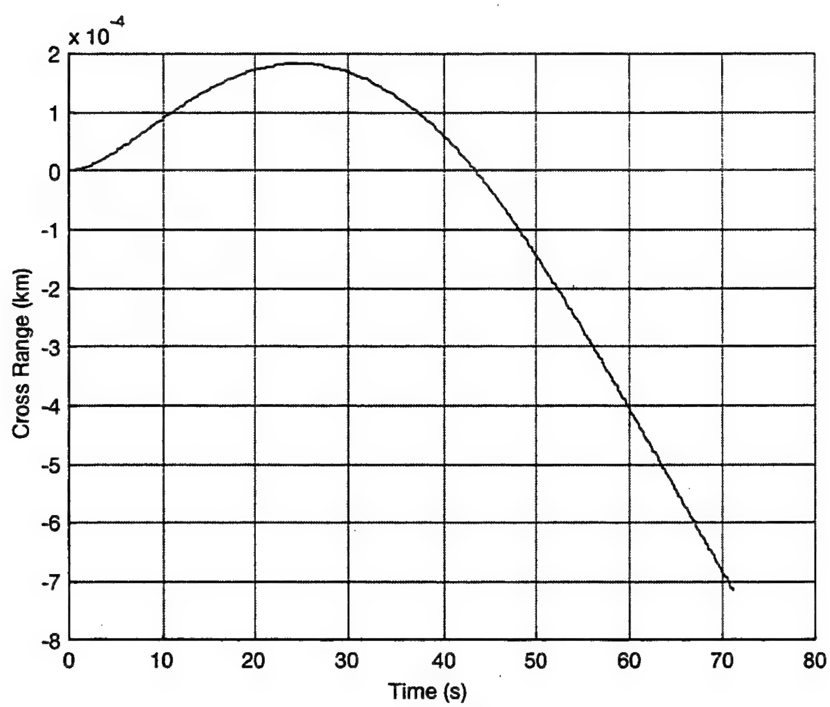

Figure 3. Cross Range.

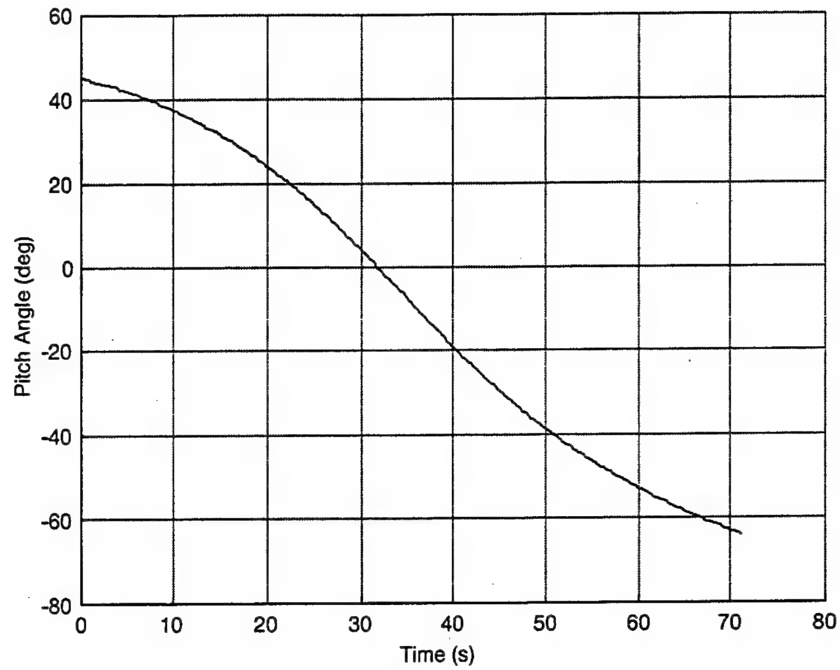

Figure 4. Euler Pitch Angle.

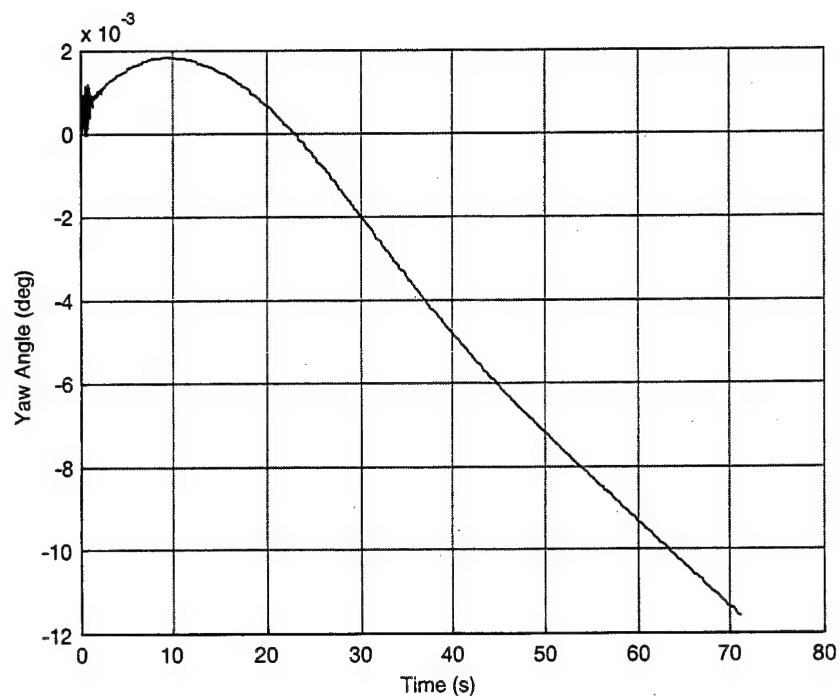

Figure 5. Euler Yaw Angle.

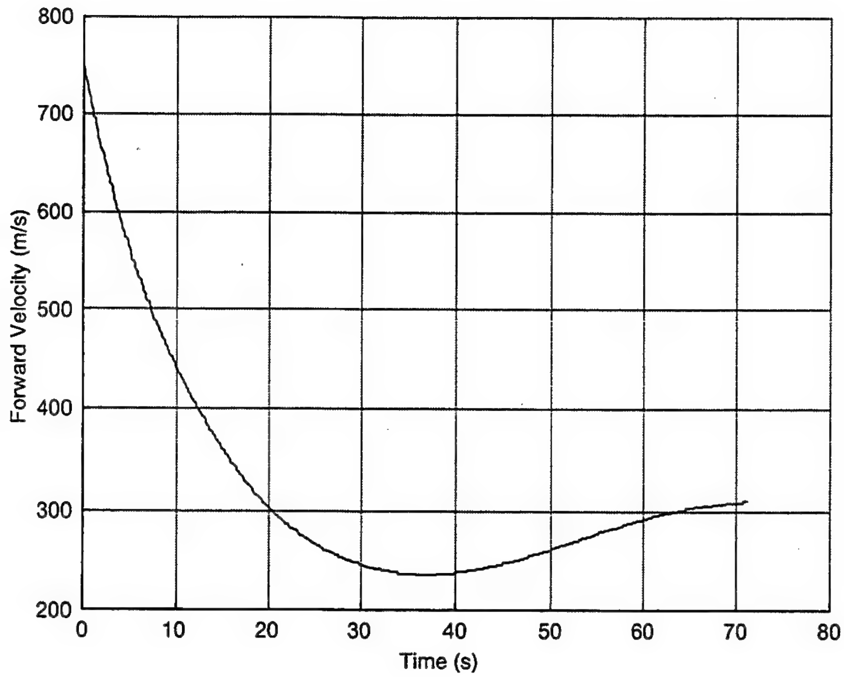

Figure 6. Forward Body Velocity.

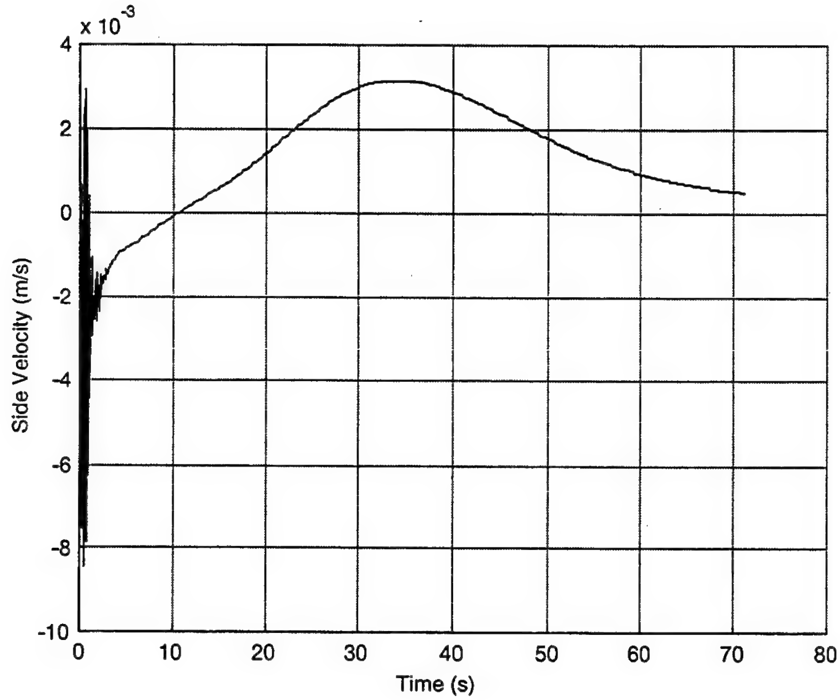

Figure 7. Side Body Velocity.

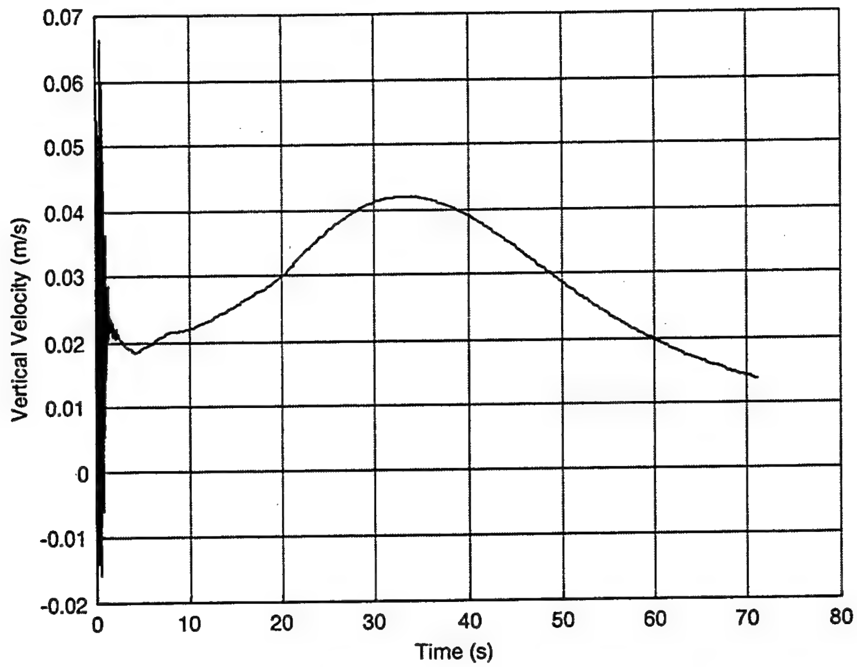

Figure 8. Vertical Body Velocity.

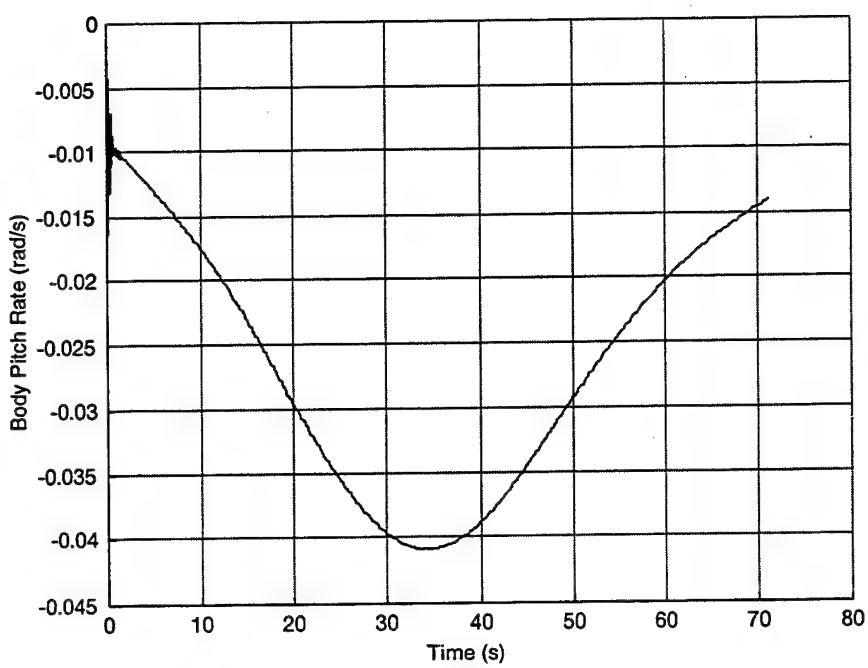

Figure 9. Pitch Rate.

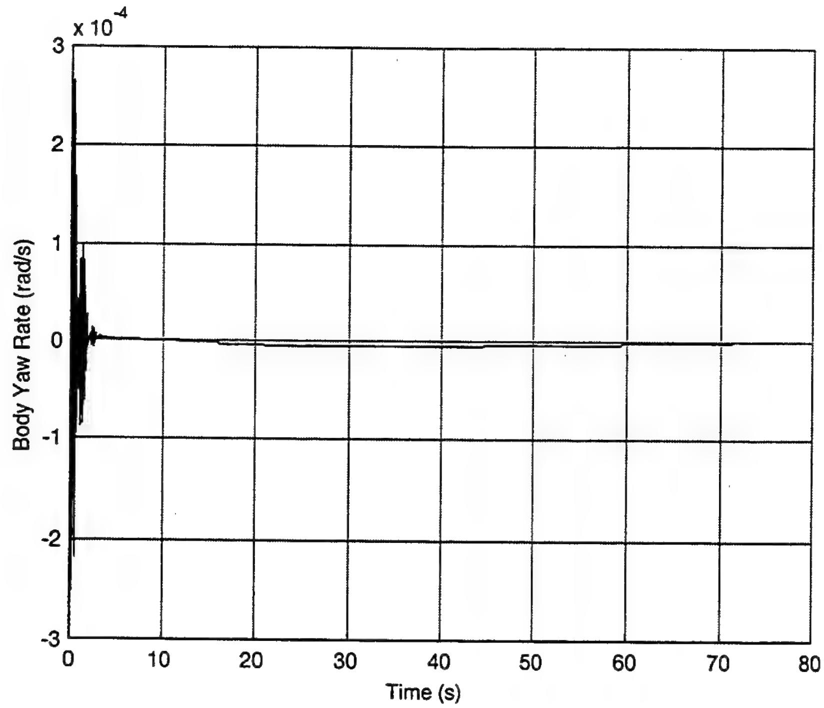

Figure 10. Yaw Rate.

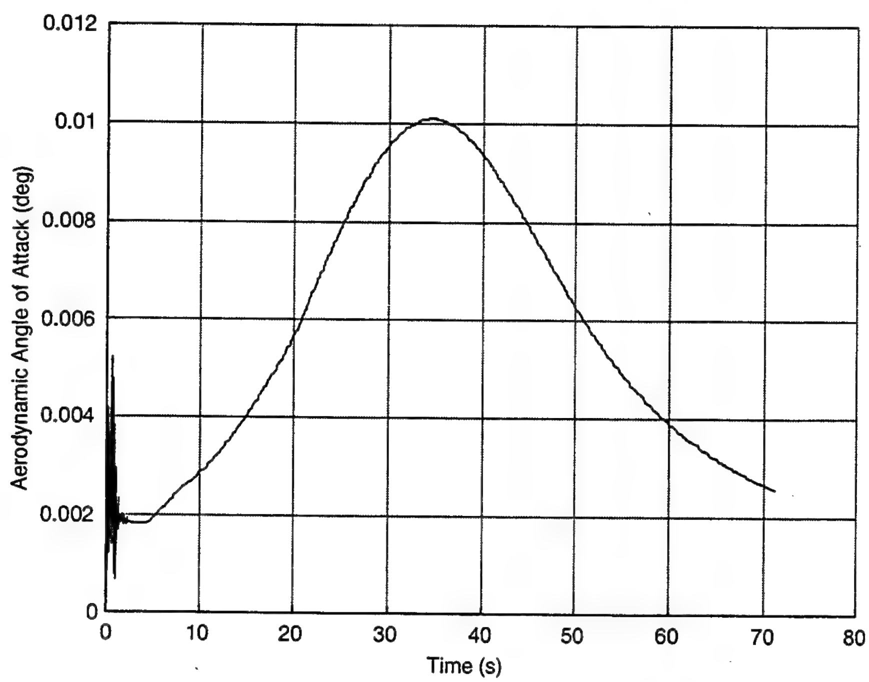

Figure 11. Aerodynamic Angle of Attack.

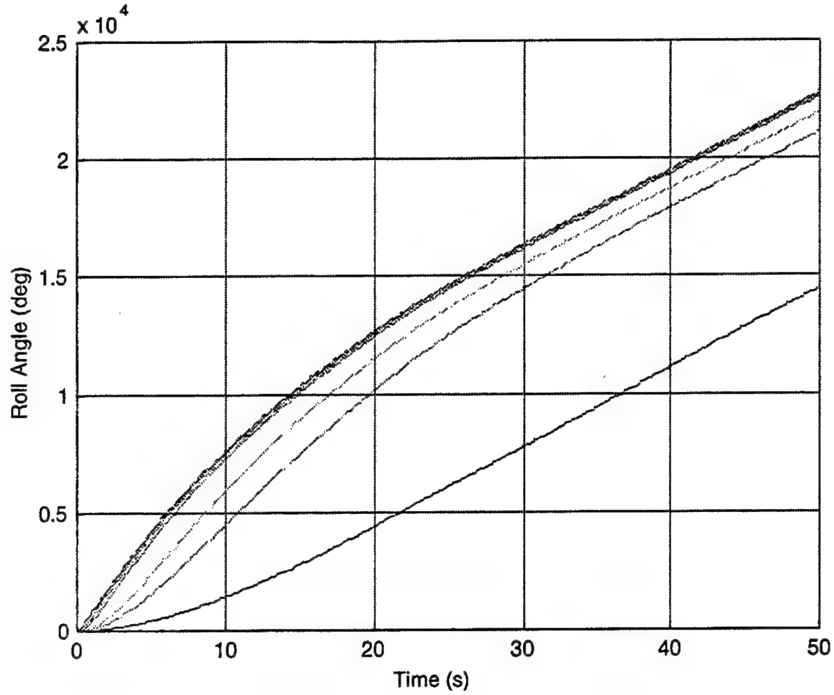

Figure 12. Roll Angle (Mass Ratio = 1%, Damping Coefficient = 0.01–0.000001).

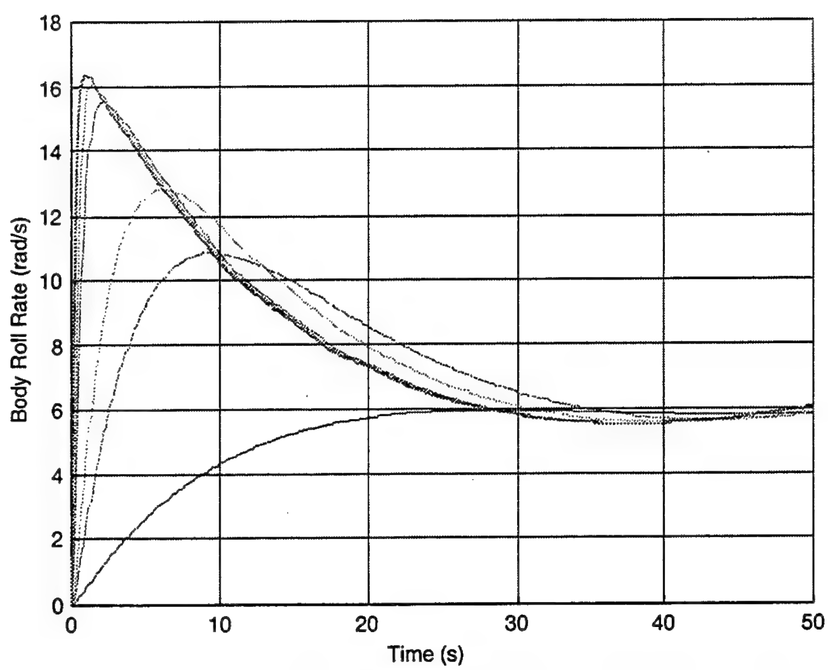

Figure 13. Roll Rate (Mass Ratio = 1%, Damping Coefficient = 0.01–0.000001).

and 0.01 ft-lbf/rps. In Figure 12, the lowest trace is the response of the aft body for the lowest value of bearing resistance. The upper trace is the forward body roll response. For a bearing resistance coefficient of 0.00005, the aft body roll response is essentially the same as the forward body since both bodies rapidly couple in the roll axis. Figure 13 shows the roll rate trace for this simulation set. It is interesting that for lower values of bearing resistance, the aft body roll rate overshoots the forward body roll rate before settling.

Figures 14 and 15 show the roll angle and roll rate response of forward and aft bodies under the same conditions as the previous case, except the mass ratio is now 50% rather than 1%. While the basic character of the roll response is the same, the aft body roll angle, and hence roll rate, build up relatively slowly due to the increase in aft body inertia.

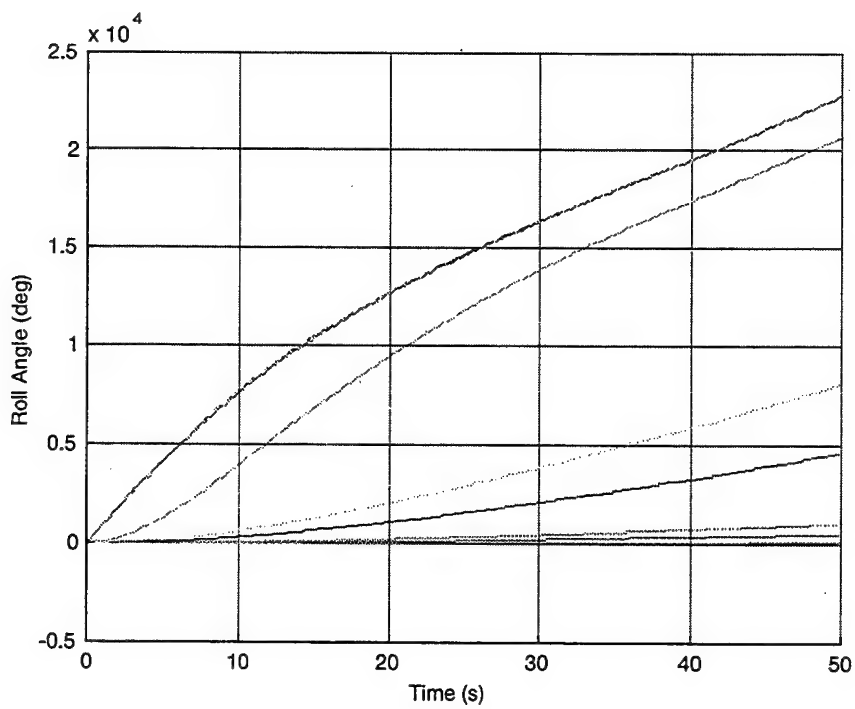

Figure 14. Roll Angle (Mass Ratio = 50%, Damping Coefficient = 0.01–0.000001).

Figures 16–19 show the response of the system under the same conditions as Figures 14 and 15, except the initial roll rate of the aft body is −100 rad/s. Like the previous simulation results,

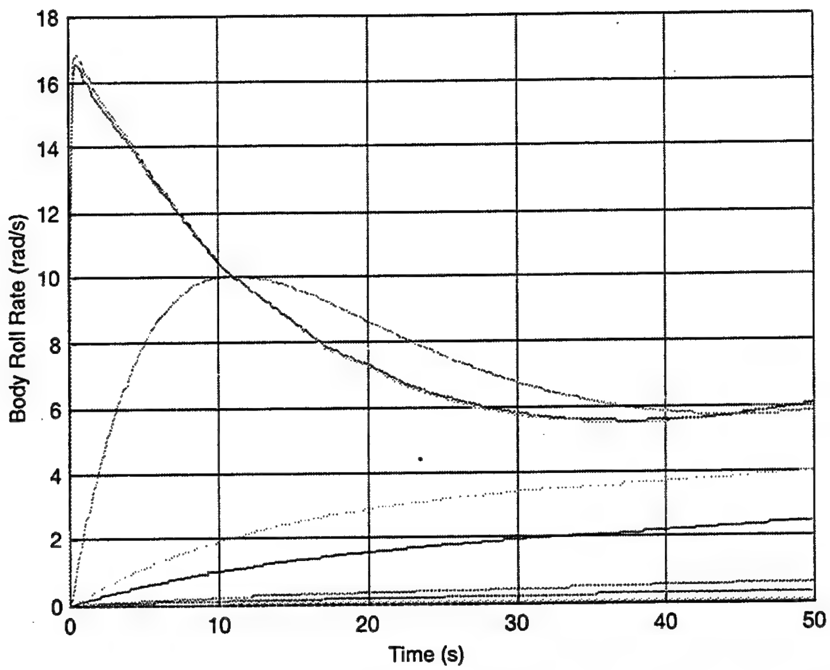

Figure 15. Roll Rate (Mass Ratio = 50%, Damping Coefficient = 0.01–0.000001).

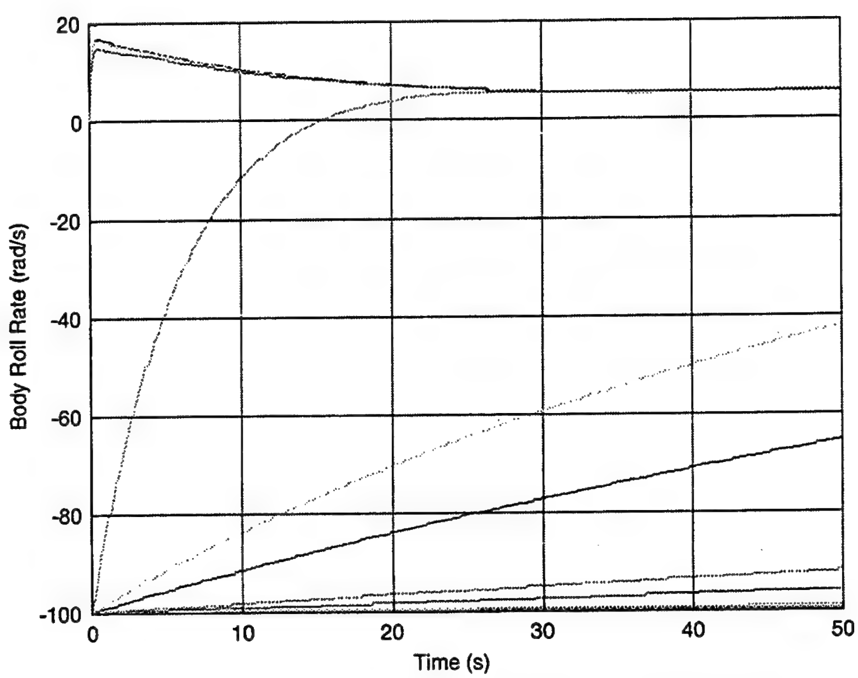

Figure 16. Roll Rate (Mass Ratio = 50%, Damping Coefficient = 0.01–0.000001).

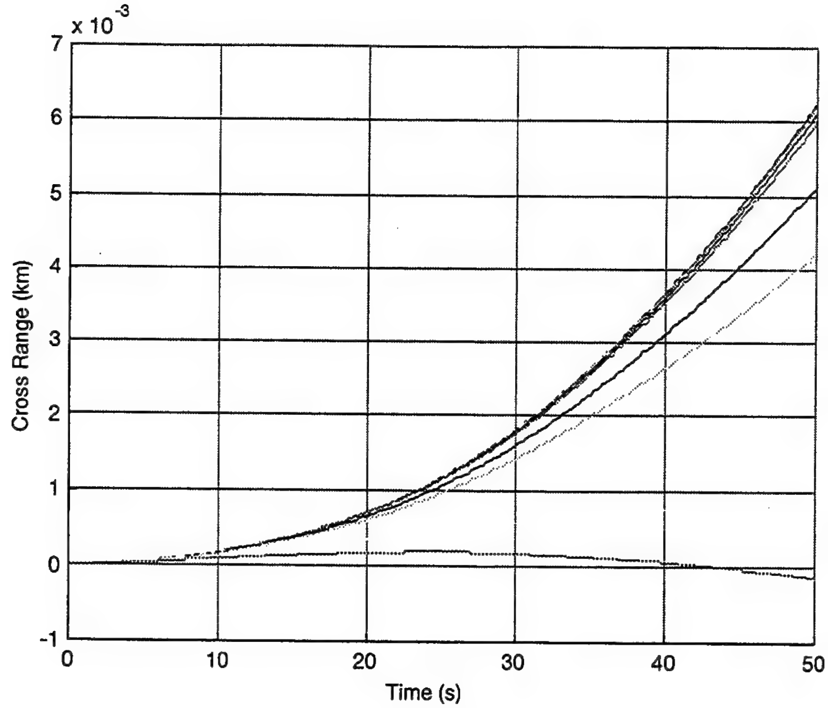

Figure 17. Cross Range (Mass Ratio = 50%, Damping Coefficient = 0.01–0.000001).

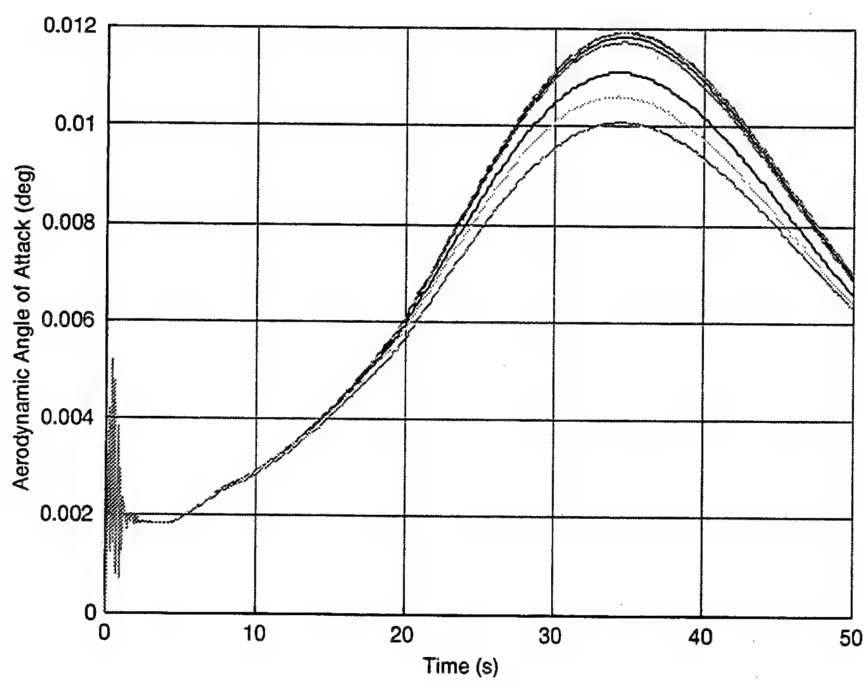

Figure 18. Angle of Attack (Mass Ratio = 50%, Damping Coefficient = 0.01–0.000001).

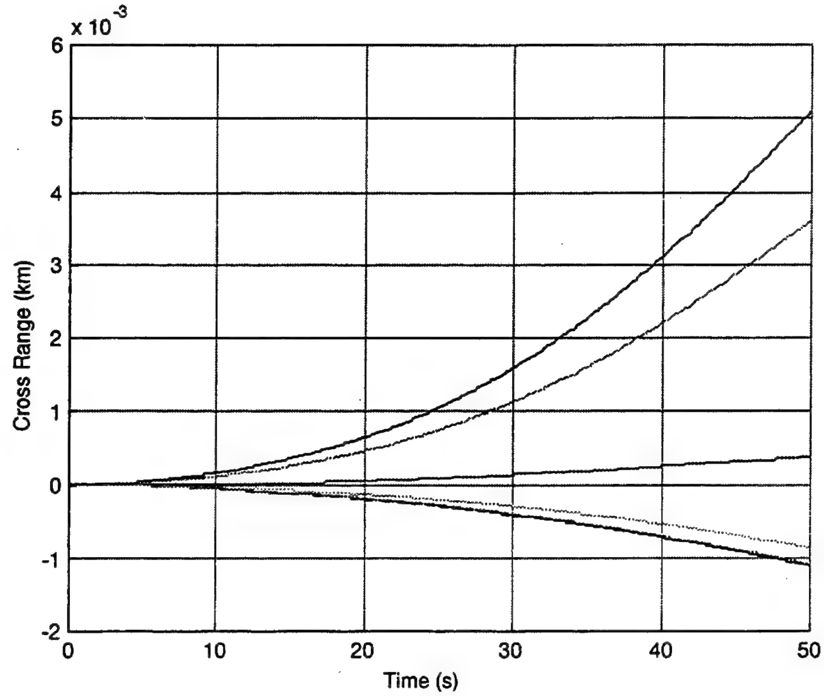

Figure 19. Cross Range (Damping Coefficient = 0.0005, Mass Ratio = 1%–50%).

lower values of bearing resistance produce slower roll response in the aft body. For larger splits in the forward and aft body roll rates, the trajectory begins to change as a function of bearing resistance owning to the fact that the roll response is sensitive to bearing resistance. In particular, Figure 17 shows the cross range under these circumstances. While the spray in the trajectory is only on the order of 10 m, it points to the fact that if the forward and aft bodies possess substantially different initial roll rates, the trajectory becomes a function of bearing resistance.

Figures 19–22 show system response under the same conditions as Figures 16–19, except now the mass ratio is varied. Figure 22 shows the roll rate response. Due to aft body inertia changes, the roll response varies significantly with mass ratio. Subsequently, the trajectory begins to vary as well. Similar to the previous case, the trajectory spray is on the order of 10 m; this shows that trajectory of configurations with forward and aft bodies operating at significantly different roll rates is a function of the mass ratio.

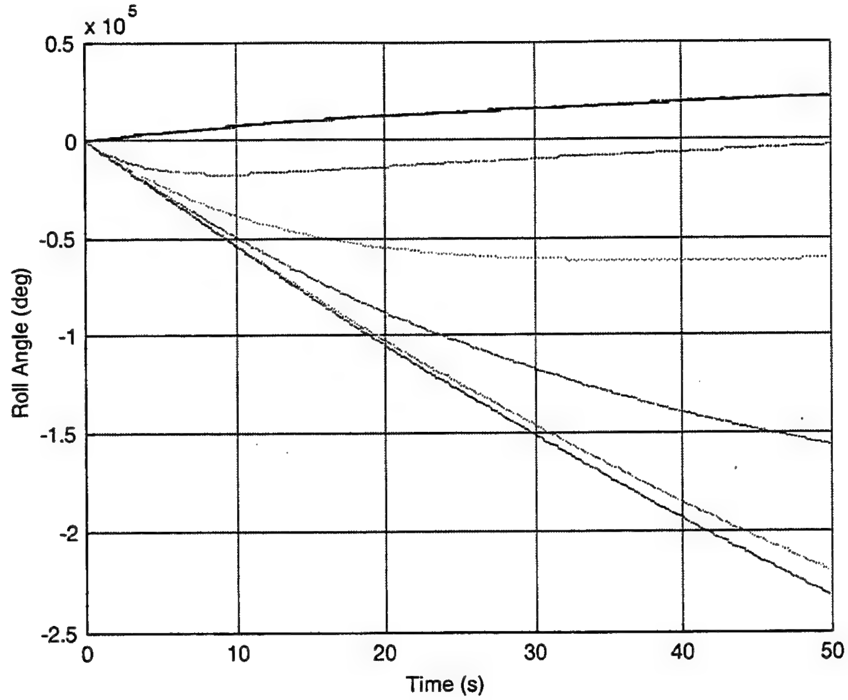

Figure 20. Roll Angle (Damping Coefficient = 0.0005, Mass Ratio = 1%–50%).

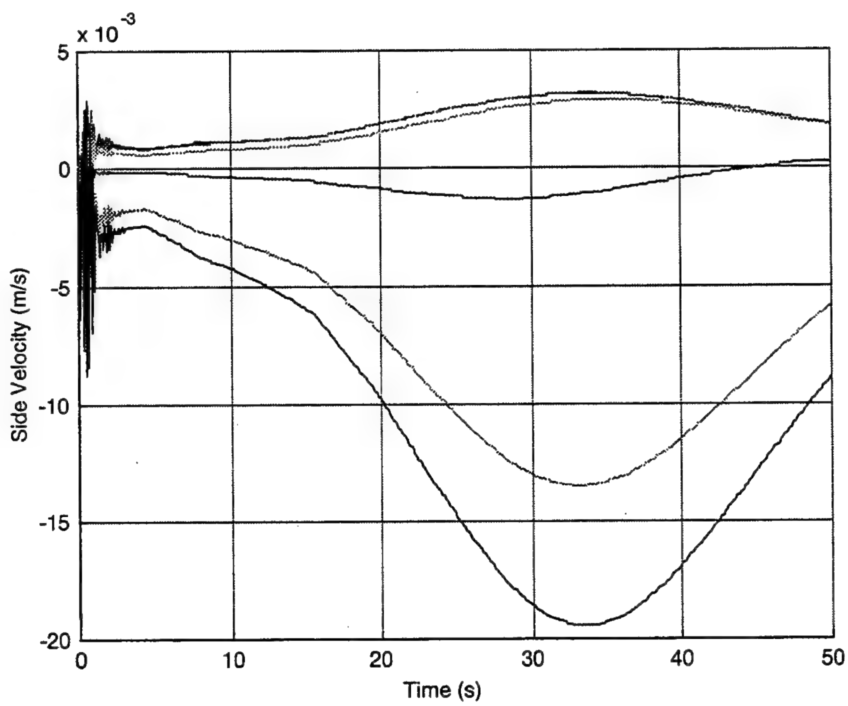

Figure 21. Side Velocity (Damping Coefficient = 0.0005, Mass Ratio = 1%–50%).

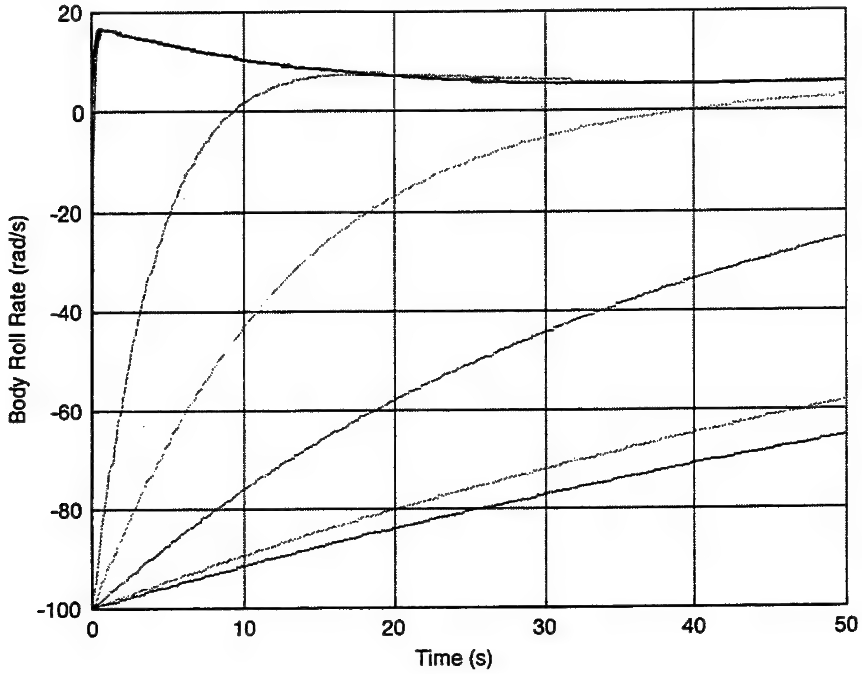

Figure 22. Roll Rate (Damping Coefficient = 0.0005, Mass Ratio = 1%–50%).

4. Conclusions

The equations of motion for a differential roll projectile configuration with seven degrees of freedom have been developed and exercised. The dynamic equations allow the forward and aft bodies to be mass unbalanced. A hydrodynamic bearing between the forward and aft components couples the roll degrees of freedom. Bearing resistance and forward/aft body mass ratio are the dominant factors in determining the roll dynamics. For spin rates typical of fin-stabilized projectiles, the trajectory is essentially independent of both bearing resistance and mass ratio. However, for configurations with the forward and aft components operating at significantly different roll rates, the trajectory depends on the mass ratio and bearing resistance.

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5. References

1. Etkin, B. *Dynamics of Atmospheric Flight.* New York: John Wiley and Sons, 1972.
2. Von Mises, R. *Theory of Flight.* New York: Dover Publications Inc., 1959.
3. Close, C. M., and D. K. Frederick. *Modeling and Analysis of Dynamic Systems.* New York: John Wiley and Sons, 1995.

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Appendix A:

Constraint Forces and Moments

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The rotation kinetic differential equations are derived by first splitting the two body system at the bearing connection point. Figures A-1 and A-2 show the external loads and internal constraint forces acting on both the forward and aft bodies, respectively.

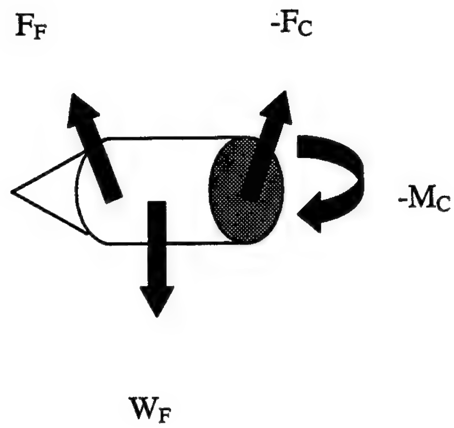

Figure A-1. Forces and Moments on the Forward Body.

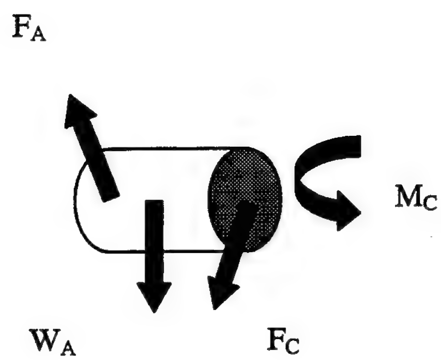

Figure A-2. Forces and Moments on the Aft Body.

The constraint force, F_C, and the constraint moment, M_C, couple the forward and aft bodies. Key to the development of the rotation kinetic differential equations is the ability to solve for the constraint forces and moments as a function of state variables and time derivatives of state variables.

An expression for the constraint force can be obtained by subtracting the translation dynamic equations for both bodies.

$$\frac{m}{m_F m_A}\vec{F}_C = \frac{\vec{F}_F}{m_F} - \frac{\vec{F}_A}{m_A} + \vec{a}_{A/I} - \vec{a}_{F/I}. \tag{A-1}$$

The acceleration of the mass center of the forward and aft bodies, $\vec{a}_{F/I}$ and $\vec{a}_{A/I}$, can be expressed in terms of the acceleration of the composite body mass center. After making this substitution, the constraint force components in the fixed-plane reference frame can be expressed in the following manner:

$$\begin{Bmatrix} F_{CX} \\ F_{CY} \\ F_{CZ} \end{Bmatrix} = [\overline{F}_F]\begin{Bmatrix} \dot{p}_F \\ \dot{q} \\ \dot{r} \end{Bmatrix} + [\overline{F}_A]\begin{Bmatrix} \dot{p}_A \\ \dot{q} \\ \dot{r} \end{Bmatrix} + \{\overline{F}_0\}. \tag{A-2}$$

The matrices $[\overline{F}_F]$, $[\overline{F}_A]$, and $\{\overline{F}_0\}$ are complicated functions of the state variables and the geometry of the configuration.

The components in the fixed-plane reference frame of the moment of the constraint force, acting on the forward body about the forward body mass center, can be written in the following manner:

$$\begin{Bmatrix} M_{FC_{FX}} \\ M_{FC_{FY}} \\ M_{FC_{FZ}} \end{Bmatrix} = [\overline{M}_{FF}]\begin{Bmatrix} \dot{p}_F \\ \dot{q} \\ \dot{r} \end{Bmatrix} + [\overline{M}_{FA}]\begin{Bmatrix} \dot{p}_A \\ \dot{q} \\ \dot{r} \end{Bmatrix} + \{\overline{M}_{F0}\}. \tag{A-3}$$

In a similar way, the components in the fixed-plane reference frame of the moment of the constraint force, acting on the aft body about the aft body mass center, can be written in the following manner:

$$\begin{Bmatrix} M_{FC_{AX}} \\ M_{FC_{AY}} \\ M_{FC_{AZ}} \end{Bmatrix} = \left[\overline{M}_{AF}\right] \begin{Bmatrix} \dot{p}_F \\ \dot{q} \\ \dot{r} \end{Bmatrix} + \left[\overline{M}_{AA}\right] \begin{Bmatrix} \dot{p}_A \\ \dot{q} \\ \dot{r} \end{Bmatrix} + \left\{\overline{M}_{A0}\right\}. \tag{A-4}$$

The matrices $[\overline{M}_{FF}]$, $[\overline{M}_{FA}]$, $[\overline{M}_{AA}]$, $[\overline{M}_{AF}]$, $\{\overline{M}_{F0}\}$, and $\{\overline{M}_{A0}\}$ are also complicated functions of the state variables and the geometry of the configuration.

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Appendix B:

Rotation Kinetic Equations

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The rotation kinetic differential equations are derived by first writing the Euler equations for the forward and aft bodies separately. These equations are expressed in the fixed-plane reference frame are general, and allow for a fully populated inertia matrix and mass unbalance. Equations (A-2), (A-3), and (A-4) are substituted into both sets of rotation kinetic equations for the forward and aft bodies. At this point, both sets of equations still have unknown constraint moments at the bearing connection point. To eliminate the bearing constraint moments in the y and z direction in the fixed-plane coordinate system, the y and z components of the rotation kinetic equations for the forward and aft bodies are added together to form two dynamic equations that are free of constraint moments. In this way, the constraint moments at the bearing have been eliminated analytically.

The forward and aft bodies are connected through a hydrodynamic bearing. The moment transmitted across a hydrodynamic bearing can be modeled as viscous damping.[1] The constitutive relation governing the constraint moment transmitted across a hydrodynamic bearing is given by Equation (B-1).

$$M_V = c_V \left(p_F - p_A \right). \tag{B-1}$$

If the viscous damping coefficient, c_V, equals zero, then the forward and aft body connection is frictionless.

The effective inertia matrix is a 4×4 matrix that is a combination of the inertia matrices of both the forward and aft bodies. As an aid in developing a formula for the effective inertia matrix, define the following intermediate matrices:

$$[I_{FF}] = [T_F]^T [I_F][T_F] + [\overline{M}_{FF}], \tag{B-2}$$

[1] Close, C. M., and D. K. Frederick. *Modeling and Analysis of Dynamic Systems.* New York: John Wiley and Sons, 1995.

$$[I_{FA}] = [\overline{M}_{FA}], \tag{B-3}$$

$$[I_{AA}] = [T_A]^T [I_A][T_A] - [\overline{M}_{AA}], \text{ and} \tag{B-4}$$

$$[I_{AF}] = -[\overline{M}_{AF}]. \tag{B-5}$$

Using Equations (B-2), (B-3), (B-4), and (B-5), elements of the effective inertia matrix can now be formed.

$$I_{1,1} = I_{FF_{1,1}}, \tag{B-6}$$

$$I_{1,2} = I_{FA_{1,1}}, \tag{B-7}$$

$$I_{1,3} = I_{FF_{1,2}} + I_{FA_{1,2}}, \tag{B-8}$$

$$I_{1,4} = I_{FF_{1,3}} + I_{FA_{1,3}}, \tag{B-9}$$

$$I_{2,1} = I_{AF_{1,1}}, \tag{B-10}$$

$$I_{2,1} = I_{AA_{1,1}}, \tag{B-11}$$

$$I_{2,3} = I_{AA_{1,2}} + I_{AF_{1,2}}, \tag{B-12}$$

$$I_{2,4} = I_{AA_{1,3}} + I_{AF_{1,3}}, \tag{B-13}$$

$$I_{3,1} = I_{FF_{2,1}} + I_{AF_{2,1}}, \tag{B-14}$$

$$I_{3,2} = I_{AA_{2,1}} + I_{FA_{2,1}}, \tag{B-15}$$

$$I_{3,3} = I_{FF_{2,2}} + I_{AA_{2,2}} + I_{FA_{2,2}} + I_{AF_{2,2}}, \tag{B-16}$$

$$I_{3,4} = I_{FF_{2,3}} + I_{AA_{2,3}} + I_{FA_{2,3}} + I_{AF_{2,3}}, \tag{B-17}$$

$$I_{4,1} = I_{FF_{3,1}} + I_{AF_{3,1}}, \tag{B-18}$$

$$I_{4,2} = I_{AA_{3,1}} + I_{FA_{3,1}}, \tag{B-19}$$

$$I_{4,3} = I_{FF_{3,2}} + I_{AA_{3,2}} + I_{FA_{3,2}} + I_{AF_{3,2}}, \text{ and} \tag{B-20}$$

$$I_{4,4} = I_{FF_{3,3}} + I_{AA_{3,3}} + I_{FA_{3,3}} + I_{AF_{3,3}}. \tag{B-21}$$

The elements of the right-hand side vector are given by Equations (B-22) and (B-23).

$$\begin{Bmatrix} g_{F_1} \\ g_{F_2} \\ g_{F_3} \end{Bmatrix} = \begin{Bmatrix} M_{FX} \\ M_{FY} \\ M_{FZ} \end{Bmatrix} - [\overline{S}_F] \begin{Bmatrix} p_F \\ q \\ r \end{Bmatrix} - \{\overline{M}_{F0}\}, \text{ and} \tag{B-22}$$

$$\begin{Bmatrix} g_{A_1} \\ g_{A_2} \\ g_{A_3} \end{Bmatrix} = \begin{Bmatrix} M_{AX} \\ M_{AY} \\ M_{AZ} \end{Bmatrix} - [\overline{S}_A] \begin{Bmatrix} p_A \\ q \\ r \end{Bmatrix} + \{\overline{M}_{A0}\}. \tag{B-23}$$

The matrices $[\overline{S}_F]$ and $[\overline{S}_A]$ in Equations (B-22) and (B-23) are given by Equations (B-24) and (B-25) as follows:

$$[\bar{S}_F] = [T_F]^T [I_F][\dot{T}_F] + [T_F]^T [S_F][I_F][T_F], \text{ and} \tag{B-24}$$

$$[\bar{S}_A] = [T_A]^T [I_A][\dot{T}_A] + [T_A]^T [S_A][I_A][T_A], \tag{B-25}$$

where

$$[S_F] = \begin{bmatrix} 0 & s_{\phi_F} q - c_{\phi_F} r & c_{\phi_F} q + s_{\phi_F} r \\ c_{\phi_F} r - s_{\phi_F} q & 0 & -p_F \\ -c_{\phi_F} q - s_{\phi_F} r & p_F & 0 \end{bmatrix}, \tag{B-26}$$

$$[S_A] = \begin{bmatrix} 0 & s_{\phi_A} q - c_{\phi_A} r & c_{\phi_A} q + s_{\phi_A} r \\ c_{\phi_A} r - s_{\phi_A} q & 0 & -p_A \\ -c_{\phi_A} q - s_{\phi_A} r & p_A & 0 \end{bmatrix}, \tag{B-27}$$

$$[T_F] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{\phi_F} & s_{\phi_F} \\ 0 & -s_{\phi_F} & c_{\phi_F} \end{bmatrix}, \tag{B-28}$$

$$[T_A] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{\phi_A} & s_{\phi_A} \\ 0 & -s_{\phi_A} & c_{\phi_A} \end{bmatrix}, \tag{B-29}$$

$$[\dot{T}_F] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -(p_F + t_\theta r) s_{\phi_F} & (p_F + t_\theta r) c_{\phi_F} \\ 0 & -(p_F + t_\theta r) c_{\phi_F} & -(p_F + t_\theta r) s_{\phi_F} \end{bmatrix}, \text{ and} \tag{B-30}$$

$$[\dot{T}_A] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -(p_A + t_\theta r) s_{\phi_A} & (p_A + t_\theta r) c_{\phi_A} \\ 0 & -(p_A + t_\theta r) c_{\phi_A} & -(p_A + t_\theta r) s_{\phi_A} \end{bmatrix}. \tag{B-31}$$

List of Symbols

x, y, z	Position vector components of the center of mass expressed in the inertial reference frame
θ, ψ	Euler pitch, yaw angles
ϕ_F	Euler roll angle of the forward body
ϕ_A	Euler roll angle of the aft body
u, v, w	Translation velocity components of the center of mass resolved in the fixed-plane reference frame
p_F	Roll axis component of the angular velocity vector of the forward body expressed in the fixed-plane reference frame
p_A	Roll axis component of the angular velocity vector of the aft body expressed in the fixed-plane reference frame
q, r	Components of the angular velocity vector of both the forward and aft bodies expressed in the fixed-plane reference frame
X, Y, Z	Total external force components on the projectile expressed in the fixed-plane reference frame
L_F, M_F, N_F	External moments on the forward body expressed in the fixed-plane reference frame
L_A, M_A, N_A	External moments on the aft body expressed in the fixed plane reference frame
m_F	Forward body mass
m_A	Aft body mass
m	Total projectile mass
$[I_F]$	Mass moment of inertia matrix of the forward body with respect to the forward body reference frame
$[I_A]$	Mass moment of inertia matrix of the aft body with respect to the aft body reference frame
$[I]$	Effective inertia matrix
D	Projectile characteristic length
C_i	Projectile aerodynamic coefficients
q_a	Dynamic pressure at the projectile mass center
α	Longitudinal aerodynamic angle of attack
β	Lateral aerodynamic angle of attack
$[T_F]$	Transformation matrix from the fixed-plane reference frame to the forward body reference frame
$[T_A]$	Transformation matrix from the fixed-plane reference frame to the aft body reference frame
c_V	Viscous damping coefficient
V	Magnitude-of-mass center velocity

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This report develops the equations of motion for a differential roll projectile configuration with seven degrees of freedom. The dynamic equations are generated generically such that the forward and aft components are mass unbalanced. A hydrodynamic bearing exists between the forward and aft components, which couples the roll degree of freedom. A simulation investigation shows that bearing resistance and forward/aft body mass ratio are the dominant factors in determining the roll dynamics. For spin rates typical of fin-stabilized projectiles, the trajectory is essentially independent of both bearing resistance and mass ratio.

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